Appendix A

AXTRUSION SUPPLEMENTARY MATERIALS

A.1 Carriage Stiffness Estimates

This section describes the steps to predict the stiffness performance of the carriage. First an accurate stiffness model of the individual air bearings is generated. Next a model of the carriage (made up of several individual bearings) is generated. Finally a model is developed that will allow forces and displacements to be applied and measured at different points on the way with respect to the carriage

A.1.1 Air Bearing Stiffness Calculations

An approximate formula for estimating a bearing's stiffness is[1]

$$K = \frac{P_s \cdot A}{2 \cdot h}, \tag{A.1}$$

where P_s is the supply pressure, A is the area of the pad, and h is the gap thickness. This estimate is used initially to approximate the size of components.

A more accurate bearing model is needed to make a more accurate carriage stiffness model. A better bearing model uses the actual load curves for each size bearing used in the carriage. These curves are available from the Newway web site (http://www.newwaybear-

1. Precision Machine Design page 583, Alexander H. Slocum, 1992, Society of Manufacturing Engineers, Dearborn Michigan.

ings.com/). The bearing load curves are approximated as a polynomial. The Newway™ 50 x 100 mm and 75 x 150 mm bearings are approximated by

$$L_{50x100} = 0.0065h^4 - 0.496h^3 + 14.598h^2 - 223.351h + 1937, \text{ and} \tag{A.2}$$

$$L_{75x150} = 0.0394h^4 - 2.515h^3 + 61.32h^2 - 786.5h + 5306.3, \tag{A.3}$$

where h is the bearing gap (lift) in microns, and L is the load capacity in Newtons. The stiffness of each air bearing is given by

$$K = -\frac{dL}{dx}. \tag{A.4}$$

Differentiating equations A.2 and A.3 with respect to x yield expressions for bearing stiffness [newtons per micron] as a function of gap height [microns]. The expressions for each size bearing are:

$$K_{50x100} = -0.0258h^3 + 1.489h^2 - 29.196h + 223.35 \tag{A.5}$$

$$K_{75x150} = -0.158h^3 + 7.543h^2 - 122.644h + 786.51 \tag{A.6}$$

Knowing the preload forces on each of the bearings allows the bearing gap and stiffness to be calculated. Section1.7 explains how to calculate the preload force on each bearing. The preload value is added to the actual load. These load values are then used to estimate the gap size by taking the inverses of equations A.2 and A.3, yielding

$$h_{50x100} = (8.0046 \cdot 10^{-12})L^4 - (3.583 \cdot 10^{-8})L^3 + (6.937 \cdot 10^{-5})L^2 - 0.0751L + 39.725\,; \tag{A.7}$$

$$h_{75x150} = (1.271 \cdot 10^{-13})L^4 - (1.096 \cdot 10^{-9})L^3 + (4.852 \cdot 10^{-6})L^2 - 0.0149L + 24.312\,. \tag{A.8}$$

Once the gap sizes are known, equations A.5 and A.6 are used to solve for the stiffness of each of the bearings. The results of this substitution are plotted in FigureA.1.

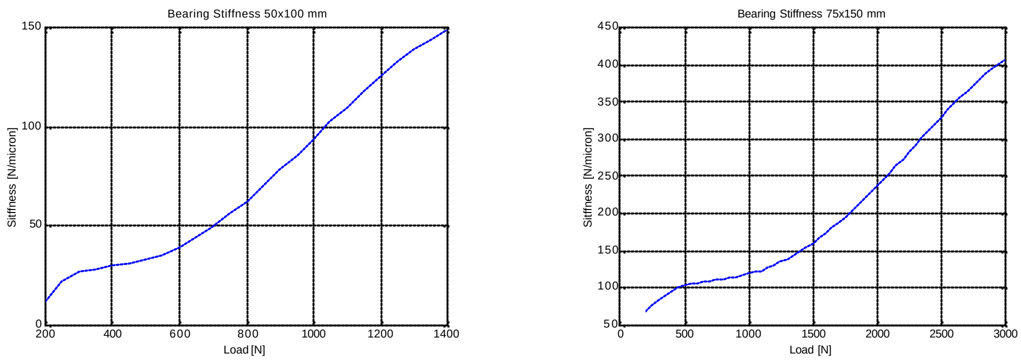

Figure A.1 A Plot of the derived bearings stiffness [Newtons/micron] vs. load [Newtons] for the Newway 50 x 100 mm (left) and 75 x 150 mm (right) air bearing running at 60 psi.

A.1.2 Estimating the Stiffness of the Axtrusion

Several assumptions are made in this analysis: 1) The actual carriage structure is infinitely stiff; all the displacement in the carriage comes from the compliance in the bearing pads. 2) The bearing stiffness is constant over the range of motion we are looking at. A compliance matrix is defined for the configuration shown below.

Figure A.2 The model used to estimate the deflection of the carriages due to tool loading forces. Each bearing was modeled as spring of constant stiffness in the direction normal to the bearing pad. The motor was modeled as spring of constant stiffness in the direction of travel.

Each pad is modeled as a spring of stiffness k_{top1}, k_{top2}, and k_{side} for the top inboard bearing pair, top outboard pair, and the side pair, respectively. The motor has a stiffness of k_{motor} in the direction of travel. A compliance matrix is calculated for this assembly.

$$\vec{C} = \begin{bmatrix} \frac{1}{k_{motor}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2k_{side}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2k_{top1} + 2k_{top2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\frac{1}{2}L_y{}^2(k_{top1} + k_{top2}) + 2k_{side}L_{zz}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\frac{1}{2}L_x^2(k_{top1} + k_{top2})} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\frac{1}{2}k_{side}L_{zx}^2} \end{bmatrix} \tag{A.9}$$

The compliance matrix is used to solve for the displacement and rotation of the carriage in response to forces and moments applied to the carriage's center of stiffness. The forces and moments applied to the center of stiffness are described by the vector

$$\vec{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix}. \tag{A.10}$$

The displacement and rotation of the carriage can be solved by

$$\overrightarrow{D_{carrage}} = \vec{C} \cdot \vec{F} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \\ \theta x \\ \theta y \\ \theta z \end{bmatrix}, \tag{A.11}$$

where $D_{carrage}$ is the displacement of the carriage (in translation and rotation).

A.1.3 Translation and Rotation of Points Not at the C.O.S.[1]

If the translation and rotation of the carriage is known, then the motion of any point fixed to the carriage can be calculated using a Homogeneous Transformation Matrix (HTM). To calculate the HTM for the displacement $D_{carrage}$ use

$$\overrightarrow{HTM} = \begin{bmatrix} C\theta y C\theta z & -C\theta y S\theta z & S\theta y & \delta x \\ S\theta x S\theta y C\theta z + C\theta x S\theta z & C\theta x C\theta z - S\theta x S\theta y S\theta z & -S\theta x C\theta y & \delta y \\ -C\theta x S\theta y C\theta z + S\theta x S\theta z & S\theta x C\theta z + C\theta x S\theta y S\theta z & C\theta x C\theta y & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \text{(A.12)}$$

where S = sine and C = cosine. To find the displacement at a point, the location of the point with respect to the coordinate system of the *HTM* must be known. This location *P* has the form

$$\vec{P} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}, \qquad \text{(A.13)}$$

where x, y, and z are the coordinates of the point with respect to the HTM. The displacement at the point is given by

$$\vec{E} = \overrightarrow{HTM} \cdot \vec{P} = \begin{bmatrix} \varepsilon x \\ \varepsilon y \\ \varepsilon z \\ 1 \end{bmatrix}. \qquad \text{(A.14)}$$

A.2 Detail Bearing Replication Steps

This is how the bearings were replicated in place in the carriage:

1. Clean and degrease the carriage pockets and way surfaces. It is important to completely remove any particles or materials that will compromise the bond between the epoxy and the carriage pockets. The way should also be cleaned

1. Precision Machine Design page 66, Alexander H. Slocum,1992, Society for Manufacuring Engineers, Dearborn Michigan

of particles and degreased so the bearings lie flat on the way and are not damaged by grit sliding between them and the way

Figure A.3 Drew Devitt (Newway Bearings) degreasing the way.

2. If the fill holes in the carriage are in the center of the pockets, then the hemispherical mounting feature in the back of each bearing should be covered with a small piece of tape. This will dramatically reduce the amount of epoxy needed to replicate each bearing in place.
3. If there are multiple inlet ports in the bearings, plug the ports that are not going to be connected to the air system with set screws or five minute epoxy. If the unused ports are not plugged then the air will not support the bearings.
4. Perform a test of the vacuum system to ensure that all the bearings can be secured to the way. Drawing a vacuum through the bearings ensures that they are aligned with the way and it prevents them from moving while the epoxy is curing.
5. Apply mold release to the linear motor coil. This will allow it to be removed from the carriage later.
6. Attach the motor coil to the carriage with the mounting screws. Draw the motor completely into the pocket. This will increase the air gap between the motor coil and magnet track from about 0.8 mm to about 3 mm, which reduces the preload force to a manageable level.
7. Attach the fixturing to the carriage.
8. Rough position the top bearings on the way.

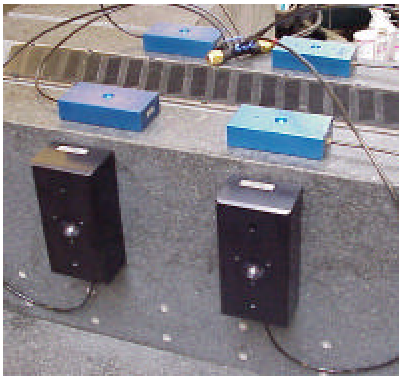

Figure A.4 Testing the vacuum system ensures that there are no leaks in the air system pior to squirting the epoxy. Notice the side bearing pads clamped to the way by the vacuum.

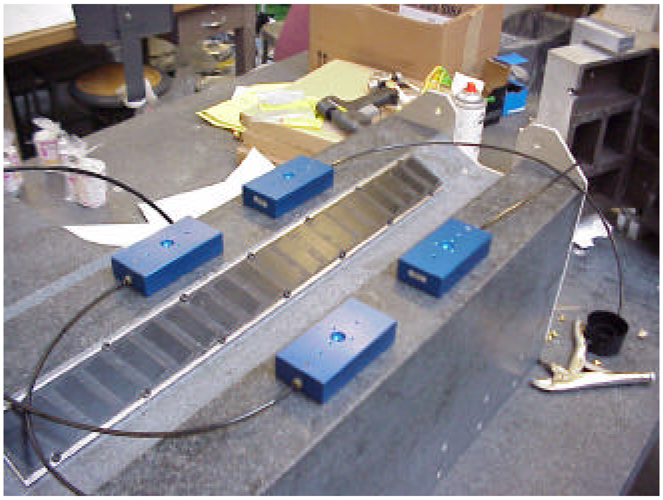

Figure A.5 The top bearings in there approximate locations on the Way.

9. Lower the carriage on the way. Fit the top bearings into their pockets.
10. Draw a vacuum through the top bearings. This holds them in place during the rest of the replication process.
11. Remove the carriage, leaving the top bearings on the way.
12. Degrease the replicating surfaces of all the bearings.
13. Place the side bearings in their pockets on the carriage.
14. Place the carriage back on the way.

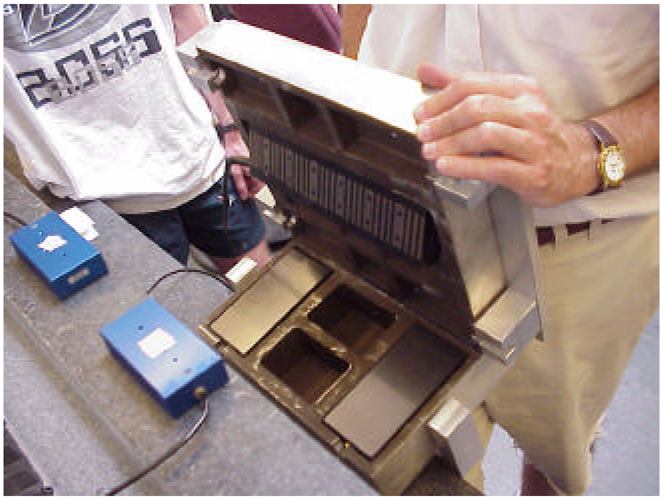

Figure A.6 The side bearings placed in their pockets before the carriage is put on the way.

15. Center the side bearings in their pockets if needed.
16. Draw a vacuum through the side bearings to hold them in place.
17. Place a piece of non-ferrous shim stock (cardboard, plastic, etc.) between the motor coil and magnet track. The shim stock's thickness should be the required air gap for the motor.
18. Lower the motor onto the shim stock and then back it off until the shim can be removed.
19. Visually inspect the air gap between the motor coil and magnet track to ensure that there is no contact between them.
20. Calculate the needed volumes of epoxy to fill each pocket. This prevents the pockets from being over filled. Overfilling could cause the epoxy to leak, and possibly even glue the carriage to the way.
21. Mix the epoxy.
22. Slowly inject the required amount of epoxy into each pocket.
23. The vacuum pump should continue to be run for about 12 hours to allow the epoxy to cure.

Figure A.7 Roger lowering the motor down onto the shim stock.

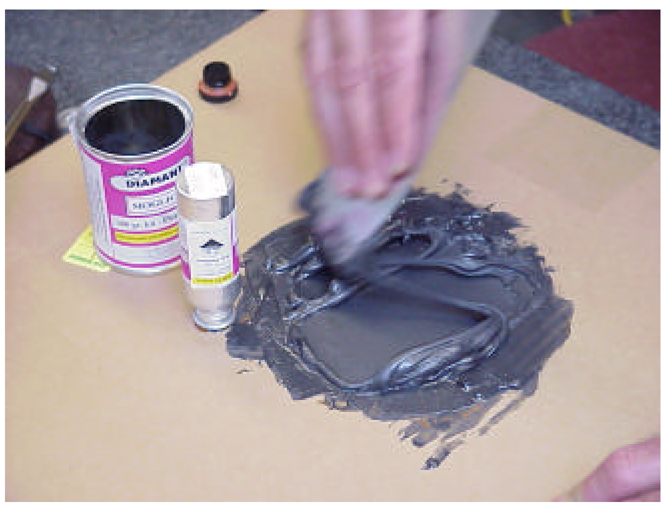

Figure A.8 The epoxy being mixed

Figure A.9 Roger injecting epoxy into one of the side pockets.

A.3 Performance Data from the Prototype

Five tests were done on the prototype to assess its performance:

- Carriage Pitch
- Carriage Yaw
- Carriage Linear Position Accuracy
- Carriage Straightness
- Carriage Stiffness

A.3.1 Carriage Pitch Data

The pitch measurements were made with a Hewlett Packard 5519A Laser System. Four data sets were taken for both pitch and yaw. The first three data sets consisted of six (6) passes, three (3) in each direction, using 320 mm of travel (the carriage has a total travel of 330 mm). The measurements were taken every 10 mm. Two data sets were run with the carriage at continuous speeds of 10 mm/s, 40 mm/s. A third data was run with the carriage stopping every 10 mm to take a measurement at rest. Finally a fourth pass was made to take measurements every 0.1 seconds, while the carriage traveled at a continuous speed of 10 mm/s. This provided a higher resolution image of what the carriage was doing in pitch. The results are summarized and plotted below.

Figure A.10 The pitch measurement setup.

TABLE A.1 Carriage Pitch Data Results

	10 mm/s	40 mm/s	10 mm/s @ 10mm increments
Raw Accuracy [arc sec.]	2.44	2.57	2.38
Raw Repeatability [arc sec.]	0.50	1.63	0.56
Raw Accuracy Forward [arc sec.]	2.44	2.57	2.32
Raw Repeatability Froward [arc sec.]	0.19	1.63	0.25
Raw Accuracy Reverse [arc sec.]	2.38	2.19	2.32
Raw Repeatability Reverse [arc sec.]	0.19	0.25	0.50

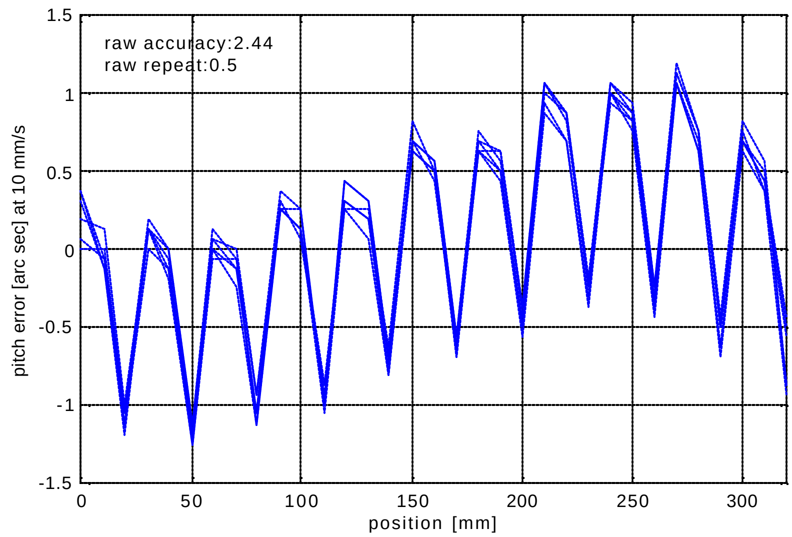

Figure A.11 Carriage Pitch [arc seconds] vs. Carriage position [mm] when the carriage is traveling at 10 mm/s. Measurements made every 10 mm. All six (6) passes are plotted.

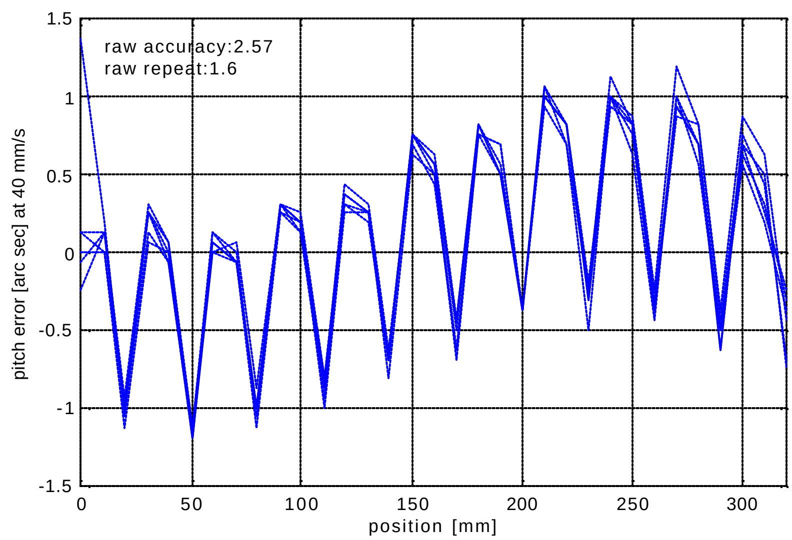

Figure A.12 Carriage Pitch [arc seconds] vs. Carriage position [mm] when the carriage is traveling at 40 mm/s. Measurements made every 10 mm. All six (6) passes are plotted.

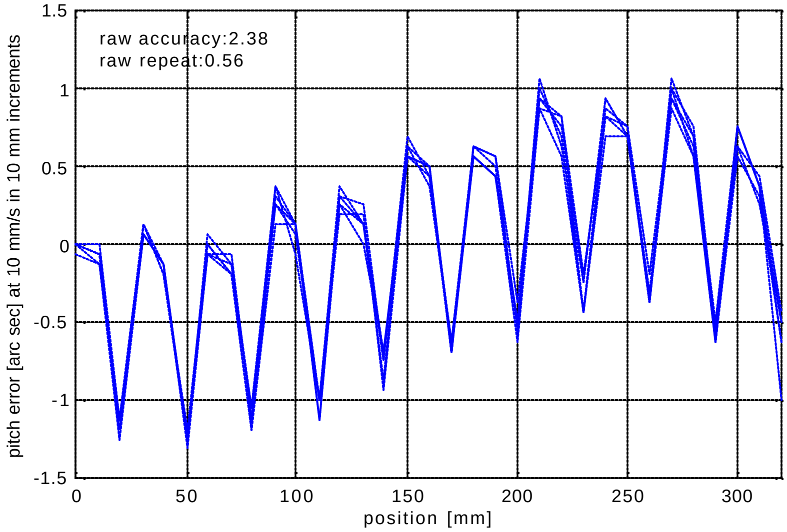

Figure A.13 Carriage Pitch [arc seconds] vs. Carriage position [mm] when the carriage is traveling at 10 mm/s stopping in 10 mm increments and the data taken after the carriage had stopped. Measurements made every 10 mm. All six (6) passes are plotted.

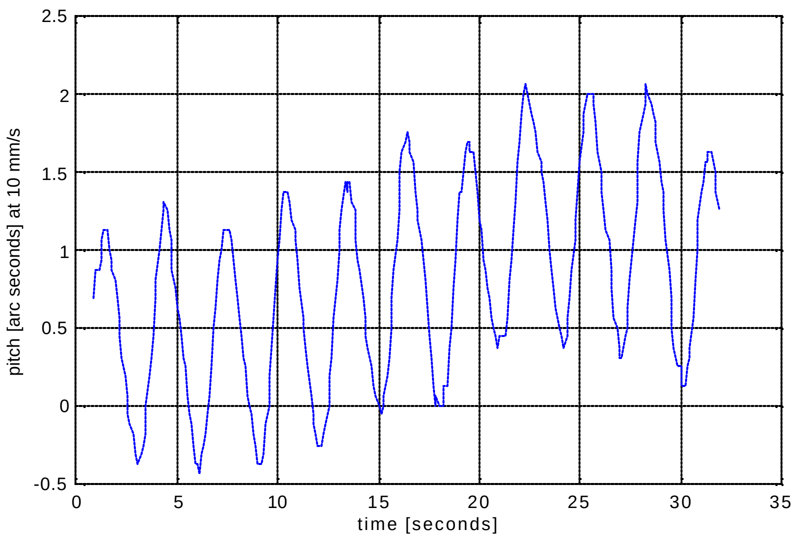

Figure A.14 Carriage Pitch [arc seconds] vs. Time [seconds]. Measurements were made every 0.1 seconds while the carriage was moving at 10 mm/s in the forward direction.

A.3.2 Carriage Yaw Data

The testing procedure for carriage yaw was identical to the testing procedure for the carriage pitch except that the inferometer was reconfigure to measure yaw. When the data was taken a very strong linear trend was observed. It is not clear if this linear trend is due to the instrumentation or an actual error in yaw. If it is an error in yaw, the linear component is trivial to remove by mapping of an orthogonal axis. If the error is an artifact of the instrumentation then the linear trend is of no concern. Data is presented in both its raw format and with the linear trend removed.

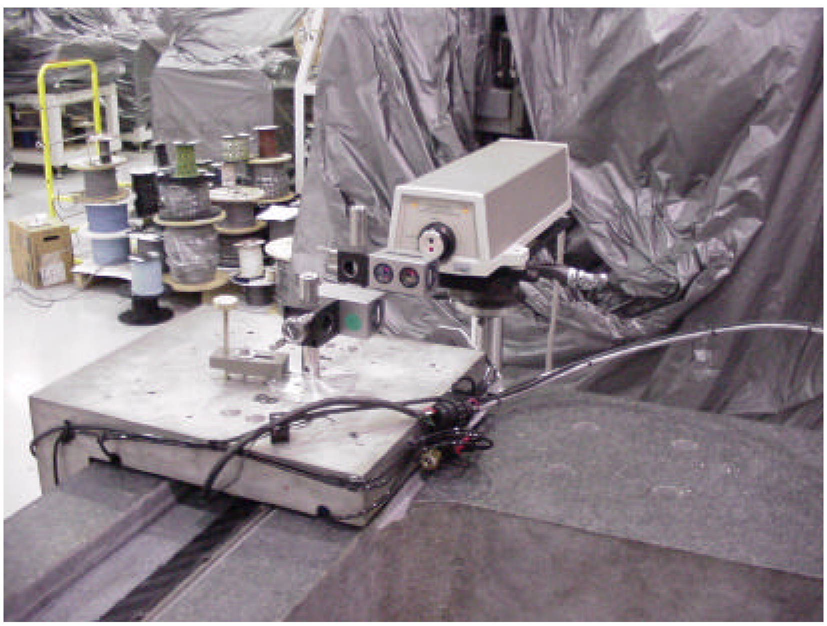

Figure A.15 The yaw measurement setup. This is identical to the pitch set up shown in FigureA.10 on page124 except the pair of inferometer lenses have been rotated 90 degrees to measure yaw instead of pitch.

TABLE A.2 Carriage Yaw Data Results

	Pitch [arc seconds]		
	10 mm/s	40 mm/s	10 mm/s @ 10mm increments
Raw Accuracy, Linear Trend Removed	1.59	1.66	1.70
Raw Repeatability, Linear Trend Removed	0.56	0.43	0.26
Raw Accuracy	6.13	6.13	6.07
Raw Repeatability	0.56	0.38	0.25
Raw Accuracy Forward	6.07	5.88	6.01
Raw Repeatability Froward	0.44	0.19	0.19
Raw Accuracy Reverse	6.07	6.13	5.94
Raw Repeatability Reverse	0.56	0.25	0.25

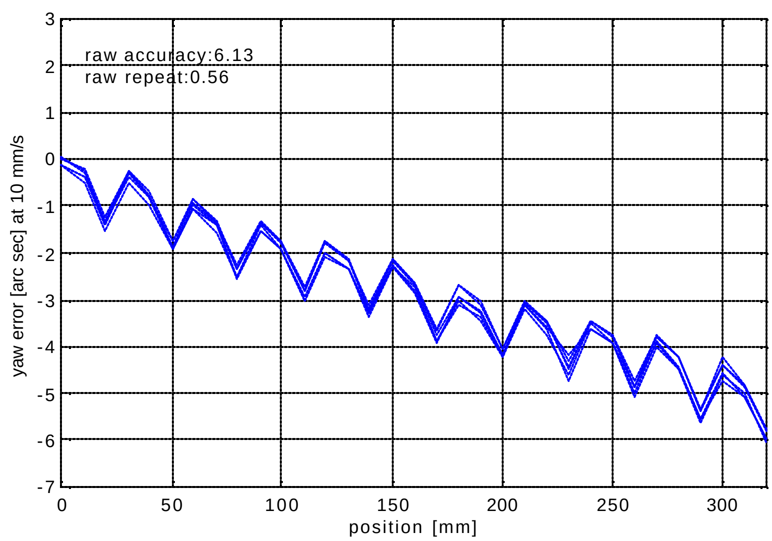

Figure A.16 Carriage Yaw [arc seconds] vs. Carriage position [mm] when the carriage is traveling at 10 mm/s. Measurements made every 10 mm. All six (6) passes are plotted

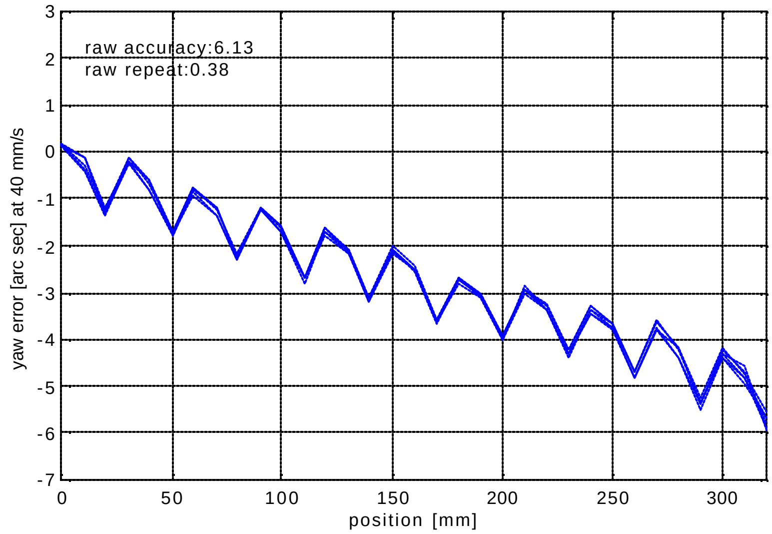

Figure A.17 Carriage Yaw [arc seconds] vs. Carriage position [mm] when the carriage is traveling at 40 mm/s. Measurements made every 10 mm. All six (6) passes are plotted

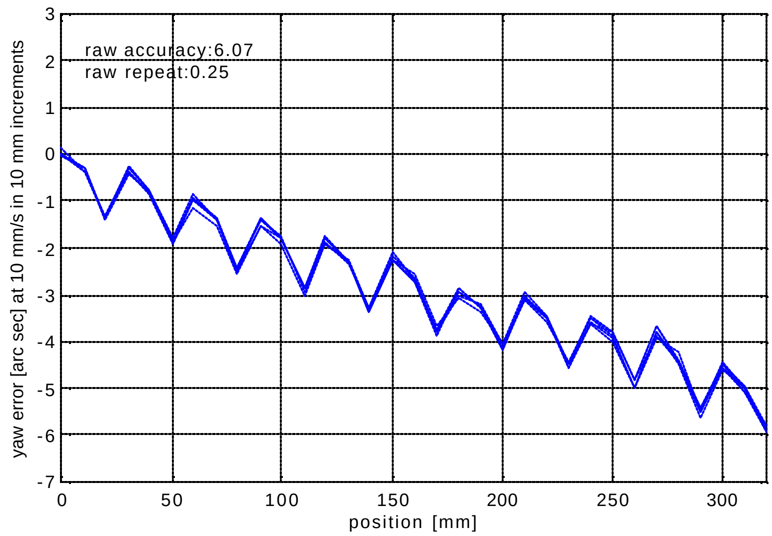

Figure A.18 Carriage Yaw [arc seconds] vs. Carriage position [mm] when the carriage is traveling at 10 mm/s stopping in 10 mm increments and the data taken after the carriage had stopped. Measurements made every 10 mm. All six (6) passes are plotted.

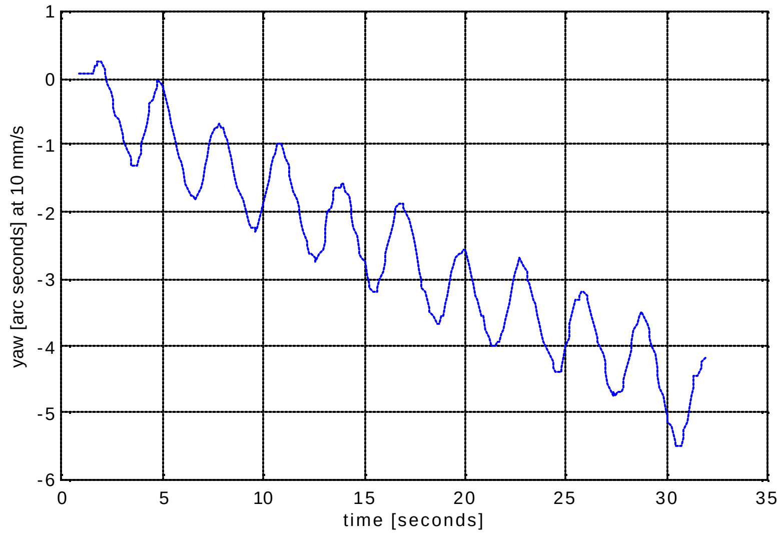

Figure A.19 Carriage Yaw [arc seconds] vs. Time [seconds]. Measurements were made every 0.1 seconds while the carriage was moving at 10 mm/s in the forward direction.

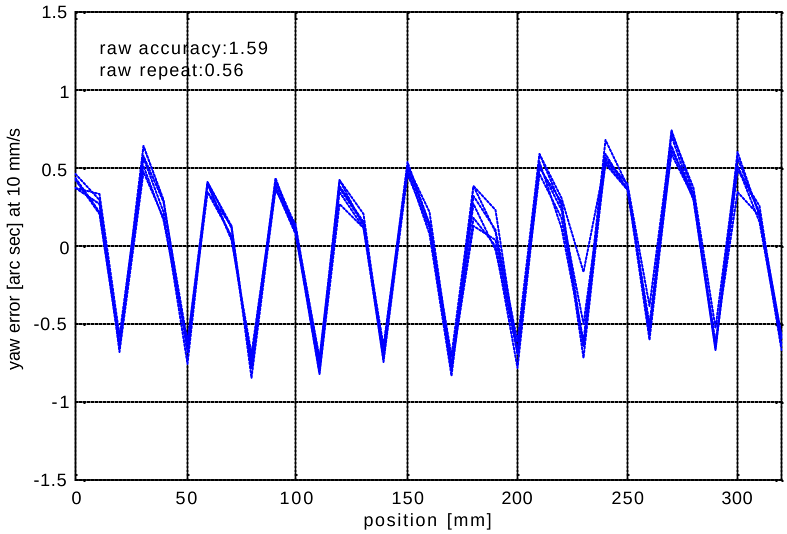

Figure A.20 Carriage Yaw [arc seconds] vs. Position [mm] with the linear trend in the data removed for the 10 mm/s test. Notice the dramatic increase in performance.

A.3.3 Linear Position Accuracy Data

This test was also done with the HP laser inferometer. The linear position accuracy was used to determine the amount of error between where the controller thought the carriage was and the carriage's actual position. The carriage was moved in 10 mm steps and its position recorded. Like the yaw data, the linear position accuracy data has a very strong linear component. If this component is removed (by the controller for example) the performance of the Axtrusion is improved by an order of magnitude. The results are summarized and plotted below.

TABLE A.3 Linear Position Accuracy Results

Raw Accuracy, Linear Trend Removed [microns]	1.34
Raw Repeatability, Linear Trend Removed [microns]	0.33
Raw Accuracy [microns]	9.808
Raw Repeatability [microns]	0.454
Raw Accuracy Forward [microns]	9.785
Raw Repeatability Froward [microns]	0.323
Raw Accuracy Reverse [microns]	9.773
Raw Repeatability Reverse [microns]	0.315

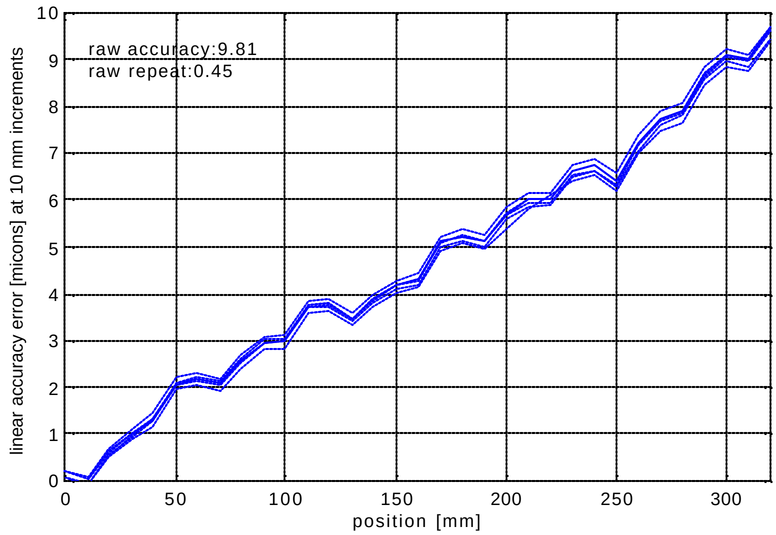

Figure A.21 Linear Position Accuracy [microns] vs. Position [mm] for the carriage. Three (3) passes in each direction are plotted.

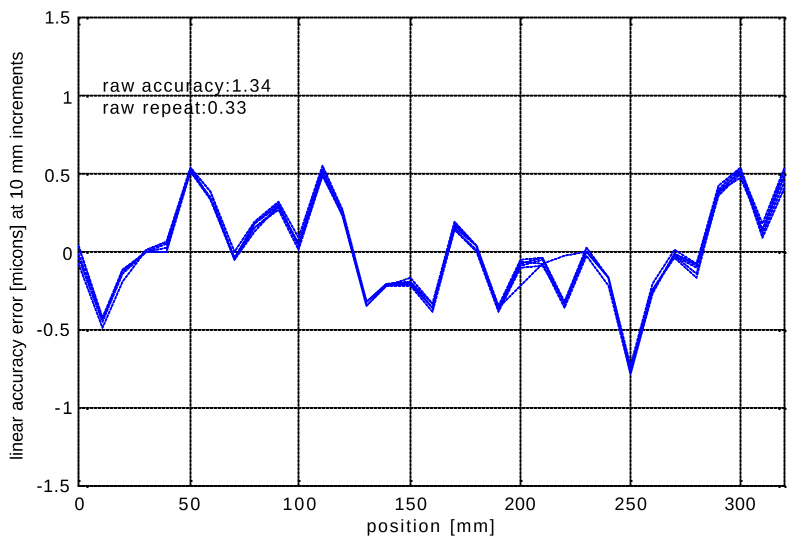

Figure A.22 Linear Position Accuracy [microns] vs. Position [mm] for the carriage, with the linear trend in the data removed. Performance is greatly increased. Three (3) passes in each direction are plotted.

A.3.4 Straightness Data

The straightness data was taken in the vertical direction only. There was not adequate fixturing to allow the measurements to be made easily in the horizontal direction. A straight edge mirror was placed on the carriage and a capacitance probe was suspended above it. As the carriage was moved the probe recorded the change in height. Since the straight edge could not be leveled perfectly the raw data would show a large linear change in the vertical position of the carriage over its length of travel. This trend was removed mathematically from the data.

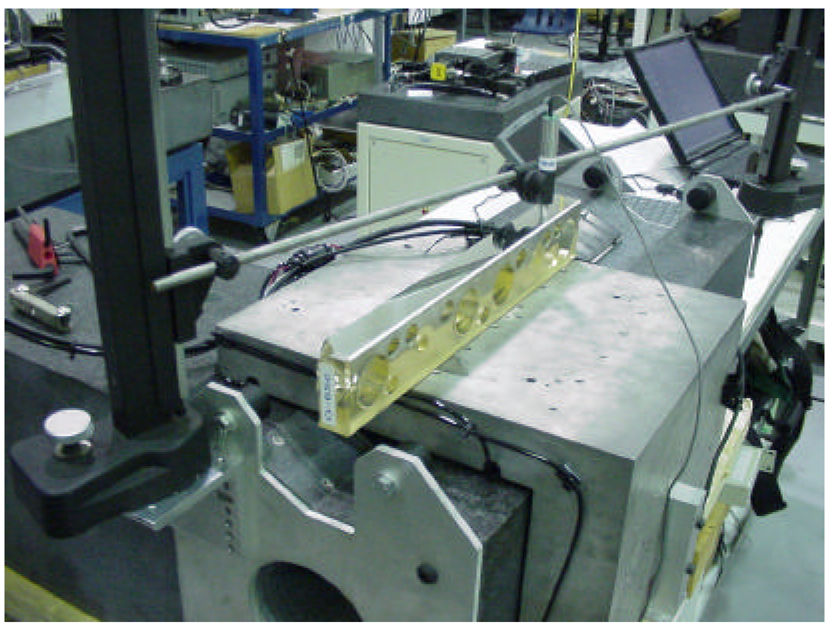

Figure A.23 The straightness measurement setup. Notice the straight edge mirror and capacitance probe suspended above it.

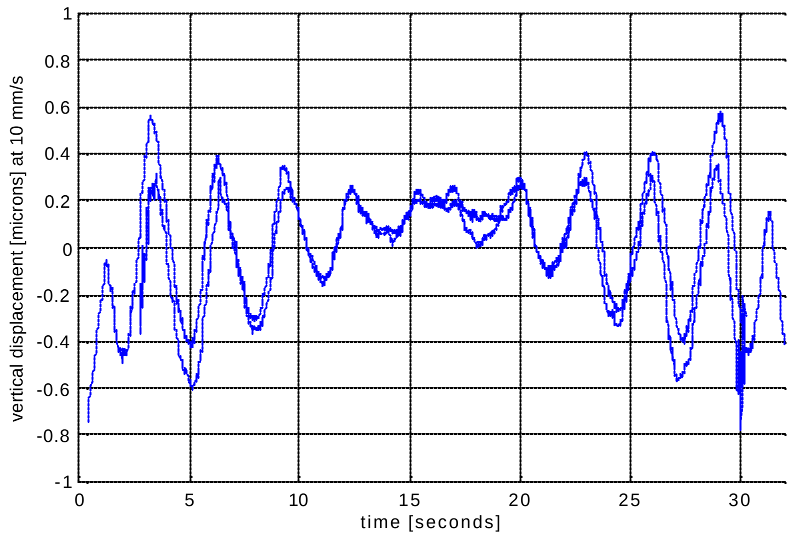

Figure A.24 Vertical Displacement [microns] vs. Time [seconds] for the carriage as it moves down the way in the forward and reverse direction. The data for the reverse direction has been flipped and shifted to show the similarity between the two curves.

A.4 The Stiffness Data

The initial stiffness measurements were fairly crude. The carriage was held in a position by the control system while dial indicator was placed on four of the points used in the modal analysis. These four points are approximately in the centers of each top bearing pads. The air bearings were run at a pressure of 4.13 Bar (60 psi). The carriage was loaded in the top center with 25 lbs and then with 50 lbs (111.2 N and 222.4 N). The displacement of each corner under both loads was recorded.

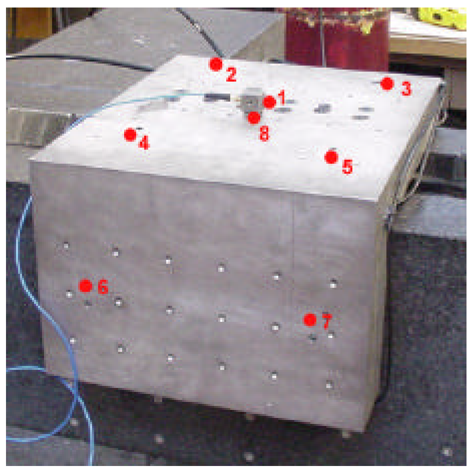

Figure A.25 Points 2, 3, 4, and 5 were used to measure the vertical displacement of the carriage when loads were applied to point 1. From this data the stiffness of the carriage was calculated.

TABLE A.4 Vertical Carriage Displacements Under Load

Carriage Location See FigureA.25	Displacement [microns] at 111.2 N (25 lbs)	Displacement [microns] at 222.4 N (50 lbs)
2	0.15	0.4
3	0.4	1.0
4	0.2	0.4
5	0.5	0.9

The approximate stiffness can be calculated from the known loads and displacements.

TABLE A.5 Vertical Carriage Stiffness Data

Carriage Location See FigureA.25	**Stiffness at 111.2 N (25 lbs) [N/micron]**	**Stiffness at 222.4 N (50 lbs) [N/micron]**	**Average Point Stiffness [N/micron]**
2	741	556	649
3	278	222	250
4	556	556	556
5	222	247	235

Therefore, the average stiffness for the carriage in the vertical direction is 422 Newtons per micron.